

Problem 5

Use the preliminary test to decide whether the following series are divergent or require further testing. *Careful:* Do *not* say that a series is convergent; the preliminary test cannot decide this.

$$\sum_{n=1}^{\infty} \frac{n!}{n! + 1}$$

Solution

Take the limit of the summand as $n \rightarrow \infty$.

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{n!}{n! + 1} &= \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n!}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n(n-1)(n-2)\cdots(3)(2)(1)}} \\ &= \frac{1}{1 + 0} \\ &= 1 \end{aligned}$$

Since it's not zero, the series diverges by the preliminary test.